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SUPPLEMENTARY INFORMATION

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SUPPLEMENTARY APPENDIX A

Distribution of Magnitudes and the Gutenberg–Richter Law

Histograms are frequently employed for preliminary inspection of empirical distributions. For a more stringent evaluation, however, contemporary statistical methodology within exploratory data analysis (EDA) utilizes the quantile–quantile (Q–Q) plot. This technique compares the empirical quantiles, data values at specified percentile positions when ordered in descending magnitude, with the corresponding quantiles of a theoretical reference distribution. In principle, one constructs a reference distribution containing an identical number of data points and directly contrasts the ordered empirical values with this idealized counterpart. Conformity to the assumed distribution is indicated when the plotted points align along a straight line (Figure 11a).

When the monthly aggregated magnitude data are sorted and compared with an ideal normal distribution, the resulting Q–Q plot exhibits a clear linear alignment (Figure 11b). Notably, this assessment incorporates all available observations without any exclusion. This outcome demonstrates that the magnitude data conform to a normal distribution.

A histogram of normally distributed random numbers yields the well-known bell-shaped curve (Figure 11c). When the same data are displayed on a semi-logarithmic scale, however, the distribution assumes a cannonball-like profile (Figure 11d). The right-hand tail appears to exhibit a linear trend, yet this is merely an artefactual pseudo-linearity: as the values increase, the slope becomes progressively steeper. This behaviour arises solely from the graphical transformation and lacks mathematical or physical significance. This is the actuality of the GR law.

Given these properties, applying the GR law to the normally distributed data in Figure 11b inevitably produces a straight line. Crucially, such an application requires discarding the majority of observations with smaller magnitudes. Within rigorous scientific practice, the selective removal of data to support a preferred interpretation constitutes cherry-picking, a form of data manipulation that is subject to criticism and must be avoided.

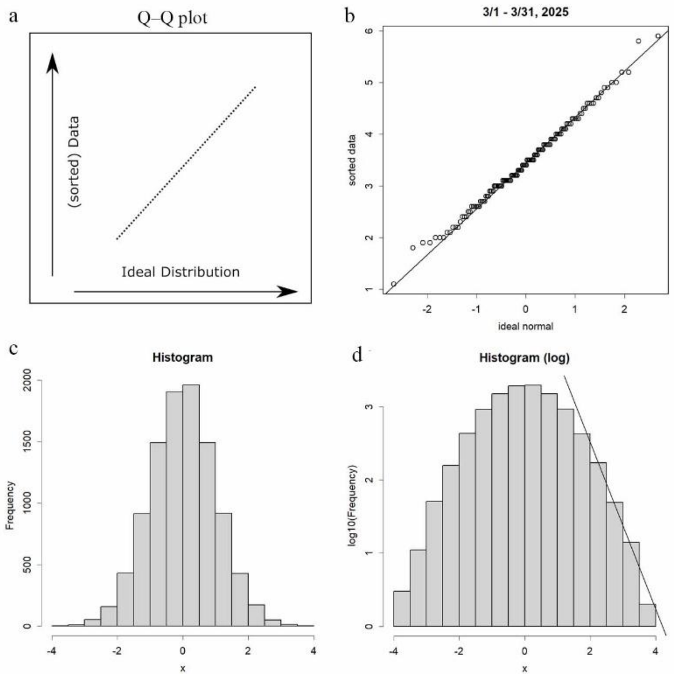

Figure 11. Quantile–quantile (Q–Q) plot: a – conceptual illustration of a Q–Q plot; b – normal Q–Q plot for the magnitude data in March 2025; c – histogram of normally distributed random numbers; d – the same histogram displayed on a semi logarithmic scale

In Figure 11a, the y-axis represents the empirical quantiles of the data, and the x-axis represents the corresponding quantiles of an ideal reference distribution. If the empirical data follow the assumed distribution, the plotted points align along a straight line.

In Figure 11b, data were obtained from the Japan Meteorological Agency (JMA), which reports the magnitudes of all perceptible earthquakes. The x-axis corresponds to an ideal normal distribution.

In Figure 11d, a straight line is fitted to the rightmost portion of the distribution, representing the slope associated with the b-value in the Gutenberg–Richter (GR) law. Because the underlying distribution cannot be linear in principle, this estimation involves substantial error. Moreover, the apparent linearity pertains only to the extreme right tail, meaning that the majority of the data, including the median, is effectively disregarded.

SUPPLEMENTARY APPENDIX B

Fundamentals of Principal Component Analysis

The theoretical background of PCA follows Jolliffe (2002) and Konishi (2015).

Principal Component Analysis (PCA) is a commonly employed technique for compressing multidimensional data. Its simplicity and clarity make it particularly suitable for scientific applications. Due to the minimal assumptions required for its computation, it consistently yields the same results regardless of the analyst, ensuring high reproducibility and objectivity.

No sample	Time	Latitude	Longitude	Depth	Magnitude
1	2025111400:0041.3	34°11.7'N	135°14.1'E	5	0.4
2	2025111400:045.2	36°36.4'N	141°1.5'E	16	1.3
3	2025111400:0424.1	39°45.2'N	143°26.3'E	14	2.7
4	2025111400:0438.1	37°56.9'N	138°9.7'E	13	0.6

Only the three dimensions Longitude, Depth, and Magnitude are used, so these are extracted.

Taking the mean value for each item gives the centre of gravity for the data. Here, we decide to rotate around this centre of gravity. Subtracting this value from all data achieves centring. This is equivalent to translating the data parallel to the rotation origin. The shape of the data remains unchanged; it remains a matrix of the same form.

$$\overline{\text{item}} = \sum \text{data}/\text{n},$$

$$\text{centered} = \text{data} - \overline{\text{item}}$$

Now, any matrix can be singular value decomposed. This is the act of decomposing the matrix into two complementary directions and their magnitudes.

$$\text{centered} = \text{U} \cdot \text{D} \cdot \text{V}^*$$

Here, U and V are unitary matrices, and V* denotes the adjoint matrix of V. Since centred is a rational number here, the adjoint matrix may be considered as the transpose matrix. The U·D denotes the matrix inner product. A unitary matrix is one that records the angle of rotation; each row and column is orthogonal, and all elements have magnitude 1. Consequently, it possesses the following property:

$$\text{U} \cdot \text{U} * = \text{U} * \cdot \text{U} = \text{I}$$

Here, I is the identity matrix. This holds because the sum of its own multiplications, i.e., its square, is 1, and its inner product with any other vector is zero. The inner product being zero with any other vector indicates that each angle is a right angle. Consequently, each vector is independent.

The magnitudes are recorded in D, which is a diagonal matrix with mostly zeros, containing the magnitudes in descending order along the diagonal. PCA involves calculating the principal components (PC) for both the sample and the item from these matrices (Konishi, 2015).

$$\text{PCsample} = \text{U}\cdot\text{D}\cdot\text{V} *\cdot\text{V} = \text{U}\cdot\text{D} = \text{centered}\cdot\text{V}$$

$$\text{PCitem} = \text{V}\cdot\text{D}\cdot\text{U} *\cdot\text{U} = \text{V}\cdot\text{D} = \text{centered} *\cdot\text{U}$$

U·D signifies assigning magnitude to a unitary matrix. Since the unitary matrix U represents orthogonal axes, it indicates the values a sample takes along those axes. This explains why PCA is described as a method for finding orthogonal axes representing the data. Alternatively, it can be viewed as rotating centred data using the unitary matrix V. This explains why PCA is described as a method for rotating data. Mathematically, both interpretations are equivalent. The PCsample represented by centred V is the rotated value of the item for each sample, while the PCitem represented by the transpose of centred V * U is the rotated value from the sample for each item. The former represents the sample's value under the new axes, while the latter represents how the item has been rotated; here, it becomes a 3 × 3 matrix.

Each is the result of a single inner product. Using R, this can be computed almost instantaneously with just a few lines of code. By now, it should be clear that there are scarcely any alternatives. If pressed, one might mention that data may sometimes be scaled as well as centred, or that the centre might be placed somewhere other than the centre of gravity (Konishi, 2015). Figure 2A shows the result of a PCsample that was also scaled; all others are results from centring alone.

PCsample represents the principal component for each sample. PCs appear as vertical vectors. PC1 is the leftmost vertical vector, with subsequent ones progressively smaller in scale as they move further to the right.

PCitem [,3]

This is R notation. [,3] denotes the third vertical vector from the left within the matrix, i.e., PC3. Incidentally, [1,] represents the first sample.

One point requires attention. In this calculation, the sign is determined randomly (the rotation direction may change by 180 degrees; it is indeterminate which direction the rotation terminates in). When displaying PC results, if inversion is preferable, multiply the corresponding PC values in both PCsample and PCitem by -1 simultaneously. Here, signs are adjusted as appropriate to avoid unnatural appearances when mapped.

PCsample [,1] <- -1*PCsample [,1]

PCitem [,1] <- -1*PCitem [,1]

PCsample represents the principal components of the sample. For instance, the values displayed in Figure 3C are these. Here, due to rotation via PCA, the Sanriku data unfolds onto the plane where PC3 = 0. Other data points move away from this plane.

The corresponding PCitem records the rotation angle. PC1 records the depth axis. For instance, at the Sanriku boundary, PCitem was:

	PC1	PC2	PC3
Longitude	44	51	-27
Latitude	-27	150	9.1
Depth	3093	0.59	0.46

PC1 is the depth axis. Since 1 unit of Longitude and Latitude roughly equals 100 km, and Depth is measured in km, the distance on the map is $(44^2 + (-27^2))^{0.5} * 100 = 3470$. Thus, when moving this distance on the map, it appears to be diving 3093 km. Therefore, the angle from sea level would be approximately $\mathrm{atan}(3093/3470)/\pi * 180 = 30°$. Similarly, PC2 indicates the azimuth on the map.

Furthermore, PC3 is perpendicular to the plane specified by PC1 and PC2. This is the norm vector of the plane. Here, the centre of gravity was

Longitude	Latitude	Depth
140.6	37.8	98.7

Therefore, the equation of this tangent plane is specified by the norm vector and a point on the plane, giving:

$-27(x - 140.6) + 9.1(y - 37.8) + 0.46(z - 98.7) = 0$

Simplifying this yields:

$y = 3.1x - 0.054 * z - 402$

Plotting this graph at z = 0 and z = -400 yields Figure 5D. Specifically, after plotting each epicentre, draw a straight line with an intercept of -402 and a slope of 3.1. The intercept at -400 km is 380. The position of the interface in Figure 3D was constructed in this manner. In reality, the length of a meridian varies with latitude, so there is some error; however, at latitudes like those in Japan, this error is likely negligible.

The distance d between two parallel lines, $ax + by + c = 0$ and $ax + by + d = 0$, can be calculated as $d = |c - d| / (a^2 + b^2)^{0.5}$. Assuming a unit of latitude is 100 km, a depth of 400 km corresponds to 100*d km, yielding a slope of 31°. This is almost identical to the value observed from PC1 (though PC1 may be affected when the surface does not submerge straight, as in Southwest boundary. Hence, this estimation is likely more accurate). The equations for each boundary were as follows:

Location	Equation	Angle
Hokkaido boundary	$y = 0.26 * x - 0.012 * z + 3.8$	42
Sanriku boundary	$y = 3.1x - 0.054 * z - 402$	31
Ogasawara boundary	$y = -1.6x + 0.018z + 255$	46
Seto boundary	$y = 0.16x - 0.011z + 12$	–
Southwest boundary total	$y = 1.5x - 0.0090z - 171$	69
Kyushu boundary	$y = 2.2x - 0.010z - 257$	80
Ryukyu Is. boundary	$y = 1.0x + 0.010z - 102$	64
Southwestern Is. boundary	$y = 0.28x - 0.0054z - 11$	63

However, the Seto boundary is shallow and the data sparse, so the angle is likely not very accurate.